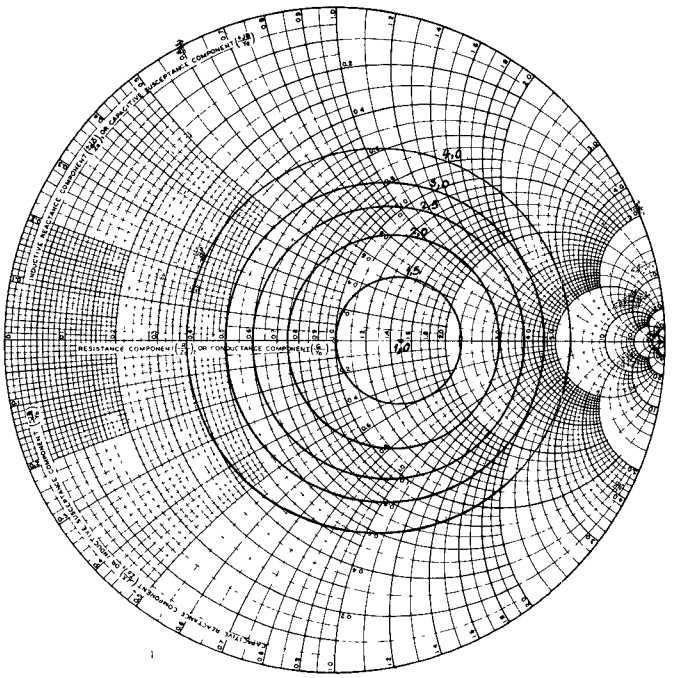

Fig. 1. Smith chart with circles of constant VSWR for an impedance of $z = 1.5$.

Consider an impedance $R + jX$, which has a constant VSWR S, with respect to an impedance $Z_1 = zZ_0$. The reflection coefficient is given by

$$\rho = \frac{R + jX - Z_1}{R + jX + Z_1}$$

therefore,

$$|\rho|^2 = \frac{(R - Z_1)^2 + X^2}{(R + Z_1)^2 + X^2} = \left(\frac{S - 1}{S + 1}\right)^2. \tag{1}$$

For any $\rho = x + jy$ to another impedance Z_0,

$$R + jX = Z_0 \frac{1 + \rho}{1 - \rho} = Z_0 \frac{1 + x + jy}{1 - x - jy}. \tag{2}$$

Separating the real and imaginary equations in (2), and substituting R and X as functions of x, y, and Z_0 into (1) yields, after considerable manipulation,

$$(x - x_0)^2 + y^2 = a^2 \tag{3}$$

in which

$$x_0 = \frac{S(z^2 - 1)}{(Sz + 1)(z + S)} \tag{4}$$

$$a = \frac{z(S^2 - 1)}{(Sz + 1)(z + S)}. \tag{5}$$

Equations (3)–(5) are seen to describe a family of circles with center points on the Smith chart real axis and offset a distance x_0 from the origin, and of radius a. These circles intersect the real-part axis of the Smith chart at points

$$r_1 = zS \qquad r_2 = z/S.$$

Thus, given *complex* values on the Z_0 chart [values of $S(Z_0)$ alone will not suffice], $S(Z_1)$ can be read off directly from the constructed circles, shown in Fig. 1 for $z = 1.5$ and various values of S.

Correction to "Efficient Calculation of Exact Group Delay Sensitivities"

J. W. BANDLER, M. R. M. RIZK, AND H. TROMP

In the above paper,[1] on page 190, the following corrections should be made.

Equation (15) should read

$$\begin{bmatrix} I_{aj}' \\ V_{bj}' \end{bmatrix} = -\begin{bmatrix} I_{aj}'^{s} \\ V_{bj}'^{s} \end{bmatrix} + \begin{bmatrix} Y_j & A_j \\ M_j & Z_j \end{bmatrix} \begin{bmatrix} V_{aj}' \\ I_{bj}' \end{bmatrix} \tag{15}$$

to match the sign convention given in Fig. 1. Consequently, (17) is

$$\begin{bmatrix} \dfrac{\partial V_{aj}}{\partial \omega} \\ \dfrac{\partial I_{bj}}{\partial \omega} \end{bmatrix} = -\begin{bmatrix} V_{aj}' \\ I_{bj}' \end{bmatrix}. \tag{17}$$

It should also be noted that in (13), the G_i' corresponds to the sensitivity component given by Bandler and Seviora,[2] but with opposite sign.

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[1] J. W. Bandler, M. R. M. Rizk, and H. Tromp, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 188–194, Apr. 1976.

[2] J. W. Bandler and R. E. Seviora, "Current trends in network optimization," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 1159–1170, Dec. 1970.